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Hybridization of isogeometric finite element method and evolutionary multi-agent system as a tool-set for multiobjective optimization of liquid fossil fuel reserves exploitation with minimizing groundwater contamination

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Abstract
In the paper we consider the approach for solving the problem of extracting liquid fossil fuels respecting not only economical aspects but also the impact on natural environment. We model the process of extracting of the oil/gas by pumping the chemical fluid into the formation with the use of IGA-FEM solver as non-stationary flow of the non-linear fluid in heterogeneous media. The problem of extracting liquid fossil fuels is defined as a multiobjective one with two contradictory objectives: maximizing the amount of the oil/gas extracted and minimizing the contamination of the groundwater. The goal of the paper is to check the performance of a hybridized solver for multiobjective optimization of liquid fossil fuel extraction (LFFEP) integrating population-based heuristic (i.e. evolutionary multi-agent system and NSGA-II algorithm for approaching the Pareto frontier) with isogeometric finite element method IGA-FEM. The results of computational experiments illustrate how the considered techniques work for a particular test scenario.

Keywords: multiobjective optimization, fossil fuels extraction, isogeometric finite element method

1 Introduction

In the real life almost any single decision has to deal with many contradictory objectives. Multicriteria optimization in the Pareto sense means the determination of all so-called non-dominated solutions in the problem space. This is difficult for most traditional approaches, and that is why for the last 25 years a variety of evolutionary multi-objective optimization techniques have been proposed [1].

At the same time real-life problems often deal with so complex phenomena that complicated expensive numerical methods, involving non-linear high order models are required. What is more, when we solve difficult inverse problems, we need not only a fast and accurate solver for

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the primal problem, but also a sophisticated methodology for solving the inverse problem itself, calling the primal problem many times to obtain the quality of solutions or gradient estimates.

Nowadays, any scientific and engineering research and works focused on reducing the human-being impact on the climate changes and reducing the greenhouse effect in particular are in the special attention.

One of the most important in this context is making the methods of natural resources and minerals exploitation not so exhaustive and destroying.

Because of the economic and social impact, an effective modeling, planing and optimizing environmental-friendly oil drilling procedures, not only maximizing the amount of the oil extracted but reducing the negative effects of the industry on our planet is the fundamental aspect.

In presented research we are modeling one of exploitation methods of liquid fossil fuel deposits consisting in pumping to the deposit of certain chemical solutions and 'sucking out' the fuel that is pushed out by the solution (so-called 'fracking'). The crucial problem in this context becomes correct placing of pumps injecting the chemical solutions to the deposit as well as the pumps sucking out the fuel.

We modeled the process of extracting of the oil/gas by pumping the chemical fluid into the formation as a non-stationary flow of the non-linear fluid in heterogeneous media. Because of the complexity of the model, a modern isogeometric finite element solver (IGA-FEM) was used [10, 11]. We defined the problem of extracting liquid fossil fuels as a multiobjetcive problem with two contradictory objectives: maximizing the amount of the oil/gas extracted and minimizing the contamination of the groundwater.

In [38] we have considered the first approach for modeling liquid fossil fuel extraction respecting not only the amount of the oil extracted but also the level of natural environment contamination. For preliminary experiments, as the inverse technique, stochastic population-based multiobjective optimization state-of-the-art algorithm i.e. NSGA-II algorithm has been used [38].

In this paper, further extended research on this topic is discussed including improved mathematical model and computational experiments for much more complicated and complex fossil fuel deposit.

Additionally, here the approach based on evolutionary computations implemented within evolutionary multi-agent system (EMAS) is evaluated. Agent-based techniques proved to be an effective alternative to classic techniques of evolutionary multi-objective optimization in a variety of different applications [31, 17].

Thus we are assessing the performance of EMAS-based reverse problem solver as a part of a hybrid with IGA-FEM solver, and the paper reports the results of computational experiments based on a well-defined test case.

The structure of the paper is as follows. First, we describe the problem to be solved. Then the foundations of the computing techniques, namely the evolutionary multi-agent system and isogeometric finite element solver, are given. A report on conducted experiments together with concluding remarks close the paper.

2 Problem formulation

The problem of extracting of the oil/gas by pumping the chemical fluid into the formation can be modeled as non-stationary flow of the non-linear fluid in heterogeneous media. The slightly more general form of time dependent problem is given by:

$$\frac{\partial u}{\partial t} - L(u) = f(\mathbf{x}, t) \quad \text{in } \Omega \times [0, T] \tag{1}$$

where $L(u)$ and f are specified later. We assume some initial state $u(\mathbf{x}, 0) = u_0(\mathbf{x})$ and impose zero Neumann boundary conditions, i.e. $\nabla u \cdot \hat{\mathbf{n}} = 0$ for $x \in \partial\Omega$. The particular case of the non-linear flow in heterogeneous media, following [12] is given by:

$$\frac{\partial u}{\partial t} - \nabla \cdot (\kappa(\mathbf{x}, u)\, \nabla u) \; = \; h(\mathbf{x}, t) \tag{2}$$

where u – pressure, κ – permeability of the medium, h – forcing arising from pumps and sinks, domain $\Omega = [0, 1]^3$. Permeability consists of two parts – static, depending on terrain properties, and the other reflecting the influence of pressure:

$$\kappa(x, u) \; = \; K_q(x) b(u) \tag{3}$$

where K_q is the prescribed formation map, and $b(u)$ is given by

$$b(u) = e^{\mu u} \tag{4}$$

We transform the time dependent problem into weak form using standard method

$$\left(v, \frac{\partial u}{\partial t}\right)_\Omega + B(v, u) = (v, f)_\Omega \tag{5}$$

where $(s, t)_\Omega$ denotes standard scalar product in $L^2(\Omega)$, i.e.

$$(s, t)_\Omega = \int_\Omega s\, t\, dx \tag{6}$$

and

$$B(v, u) = -\,(v, L(u))_\Omega \tag{7}$$

We use different methods for temporal and spatial discretization. Time is discretized using forward Euler scheme:

$$\frac{\partial u}{\partial t} \approx \frac{u_{t+1} - u_t}{\Delta t} \tag{8}$$

which leads to:

$$\left(v, \frac{u_{t+1} - u_t}{\Delta t}\right)_\Omega + b(v, u) = (v, f)_\Omega \tag{9}$$

Explicit formula for the solution in the next time step is thus

$$(v, u_{t+1})_\Omega \; = \; (v, u_t)_\Omega + \Delta t \; ((v, f)_\Omega - b(v, u)) \tag{10}$$

The problem (10) is equivalent to a sequence of isogeometric L^2-projection problems.

Pumps and sinks can be located arbitrarily inside the domain. Both pumps and sinks affect directly only a small region in their vicinity – pumps provide a constant increase of pressure, while sinks decrease pressure proportionally to the current pressure. In more detail, pumps and sinks define forcing h as follows. Let us define an auxiliary function

$$\theta_{r,R}(t) = \begin{cases} 1 & \text{for } t < r \\ \left(\frac{t-r}{R-r} - 1\right)^2 \left(\frac{t-r}{R-r} + 1\right)^2 & \text{for } r \le t \le R \\ 0 & \text{for } t > R \end{cases} \tag{11}$$

for some constants R, r (in our case, $r = 0$, $R = 0.15$ – let us denote $\theta = \theta_{0,0.15}$). Function θ assumes value 1 at $t = 0$ and falls smoothly to 0 at $t = r$. For each pump $p \in P$ and sink $s \in S$ let x_p and x_s denote its position, respectively. Forcing h is computed as

$$h(x,t) = \sum_{p \in P} \theta\left(\|x - x_p\|\right) - \sum_{s \in S} u(x,t)\,\theta\left(\|x - x_s\|\right) \tag{12}$$

that is, a pump or sink affects area around it in a radius r, and sink draining strength depends on pressure. The total amount D of drained liquid is calculated as a time integral of draining part in the above equation, i.e.

$$D = \sum_{s \in S} \int_0^T u(x,t)\,\theta\left(\|x - x_s\|\right)\,\mathrm{d}t \tag{13}$$

Groundwater region Ω_G is defined as:

$$\Omega_G = \{\mathbf{x} = (x,y,z) \colon z < 0.2\} \tag{14}$$

Contamination is computed as an integral of u in that region at the end of the simulation, that is

$$C = \int_{\Omega_G} u(x,T)\,dx \tag{15}$$

On a very high level, the problem undertaken can be formulated as follows: where the pumps injecting the chemical solutions to the formation as well as where the pumps sucking the shale gas should be located in the formation to ensure maximum volume of extracted fuel (maximum gain) and minimum contamination of the groundwater (minimum lost).

We gain a classical multiobjective optimization problem with two contradictory objectives.

From the mathematical point of view, multi-objective (or multi-criteria) optimization problem (MOOP) is formulated as follows ([3, 2, 1]):

$$MOOP \equiv \begin{cases} \text{Min/Max:} \;\; f_l(\bar{x}), \quad l = 1,2\ldots,L \\ \text{Taking into consideration:} \\ \;\; g_j(\bar{x}) \geq 0, \quad j = 1,2\ldots,J \\ \;\; h_k(\bar{x}) = 0, \quad k = 1,2\ldots,K \\ \;\; x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1,2\ldots,N \end{cases}$$

The set of constraints, both: equalities ($h_k(\bar{x})$), as well as inequalities ($g_j(\bar{x})$), and constraints related to the decision variables, i.e. lower bounds ($x_i^{(L)}$) and upper bounds ($x_i^{(U)}$), define so called searching space—feasible alternatives ($\mathcal{D}$).

In our case, the multiobjective optimization of **L**iquid **F**ossil **F**uel **E**xtraction **P**roblem (**LFFEP**) respecting the environmental impact can be formulated as follows:

$$MOOP = LFFEP \equiv \begin{cases} \text{Max: } D = \sum_{s \in S} \int_0^T u(x,t)\,\phi\left(\|x - x_s\|\right)\,\mathrm{d}t \quad \text{(amount of drained liquid)} \\ \text{Min: } C = \int_{\Omega_G} u(x,T)\,dx \quad \text{(contamination)} \\ \text{Taking into consideration:} \\ \;\; D \geq 0 \;\text{ and } C(T) \geq 0 \\ \;\; 0 \leq x_i \leq 1, \quad i = 1,2,3 \end{cases}$$

In the course of this paper multi-objective optimization in the Pareto sense is considered, so solving defined problem means determining of all feasible and non-dominated alternatives from the set ($\mathcal{D}$). Such defined set is called Pareto set ($\mathcal{P}$) and in objective space it forms so called Pareto frontier ($\mathcal{PF}$).

3 Evolutionary multi-agent system

Evolutionary computation has been successfully used for solving difficult problems and are particularly useful when classical computational methods prove ineffective. An evolutionary algorithm works properly if the population consists of fairly different individuals, i.e. the so-called diversity in the population is preserved. Loosing the population diversity limits the possibilities of the application in some areas such as multi-objective or multi-modal optimisation.

The above-described situation is related to the fact that the model of evolution employed by simple evolutionary algorithms lacks many important features observed in organic evolution [32]. This includes dynamically changing environmental conditions, neither global knowledge nor generational synchronisation assumed, co-evolution of species, evolving genotype-fenotype mapping, etc. That is why many variations of classical evolutionary algorithms have been proposed, introducing additional mechanisms following the most important phenomena in evolutionary biology e.g. dedicated cooperation mechanisms [30], coevolutionary mechanisms [37, 36, 35, 39], hierarchical approaches [23] or converting problems into multiobjective optimization problems [33]. Yet still obtained results have been not satisfying in many cases.

One may notice that during last decades the concept of software agents have gained a lot of attention [40]. This is because of intelligent interactions, which constitute the essence of multi-agent systems (MAS), and thus multi-agent systems are ideally suited for representing problems that have many solving methods, involve many perspectives and/or may be solved by many entities. As a result, agents play a key role in the integration of different techniques, which often leads to hybrid design of modern intelligent systems [19].

Since evolutionary algorithms are distributed by nature and since agents are able to perform many complex operations it was then natural that the idea of hybridization of evolutionary computations with (multi)agent systems arouse. This kind of multi-agent systems would be a computational rather then information system and requires different approach to design and implementation [20, 18]. In most such applications reported in literature (see e.g. [21] or [22] for a review) an evolutionary algorithm is used by an agent to aid realisation of some of its tasks.

But when we think about constituting a new hybrid evolutionary-agent computational paradigm in fact two approaches are possible. In the first one agents constitute a management infrastructure for a distributed realisation of an evolutionary algorithm [34]. In such an approach each agent has the population of individuals inside of it, and this sub-population is evolving according to one of (classical) evolutionary algorithm. Agents themselves can migrate within the computational environment, from one computational node to another, trying to utilize in a best way free computational resources.

Yet, as it was said, since evolutionary processes are decentralised by nature one may imagine the incorporation of evolutionary processes into a multi-agent system at a population level. It means that apart from interaction mechanisms typical of MAS (such as communication), agents are able to *reproduce* (generate new agents) and may *die* (be eliminated from the system).

Such an idea with agents located in fixed positions on some lattice (like in a cellular model of parallel evolutionary algorithms) was developed by e.g. [24]. This approach yet interesting was disregarding important, powerful and crucial in facts features of agents i.e. their autonomy and mobility.

The full realization of the idea of incorporating evolutionary processes into a multi-agent systems at a population level regarding full autonomy of agents was the decentralised model of evolution employed by an *evolutionary multi-agent system* – EMAS [25].

Agents of EMAS represent or generate solutions for a given optimisation problem. Inheritance is accomplished by an action of reproduction with the use of variation operators, like in classical evolutionary algorithms. Agents are located on islands, which constitute their local environment where direct interactions may occur, and may represent a distributed structure of computation.

Agents are able to change their location, which allows for diffusion of information and resources all over the system. Assuming that no global knowledge is available and the agents are autonomous, it is difficult to process the agents in generations. That is why a distinctive mechanism of EMAS is selection, which is realised asynchronously based on non-renewable resources [17].

In order to realize the selection process "better" (what means that they simply better solve the given problem) agents are given more resources from the environment (or from other agents) and "worse" agents are given less resources (or should give some of its resources to "better" agents).

Such mechanisms result in decentralized evolutionary processes in which individuals (agents) make independently all their decisions concerning reproduction, migration, interactions with other agents, etc., taking into consideration conditions of the environment, other agents present within the neighborhood, and resources possessed.

4 Computational method

To solve defined problem of multiobjective optimization of liquid fossil fuels extraction we proposed hybridization of evolutionary multi-agent system adapted for solving multiobjective optimization problems with dedicated solver for primal/inverse problems i.e. modern isogeometric finite element method solver IGA-FEM [8, 9, 14].

Evolutionary multi-agent system for multiobjective optimization is responsible for approaching the set of non-dominated solutions (i.e. Pareto set and Pareto frontiers respectively) with appropriately defined agents' actions whereas IGA-FEM solver is responsible for simulating distribution of chemicals in the geological formation and computing the amount of extracted fuel and contamination of groundwater depending on the localization of pumps and sinks.

Every single agent represents a location of pump(s) and sink(s) encoded in their "DNA" as the matrix of coordinates. Agents, to evaluate their solution launch IGA-FEM solver which is used here as a black box, with the input describing the location of the pumps (where the chemicals are pushed into the formation) and the sinks (where the oil is extracted). The IGA-FEM solver provides the amount of extracted oil and the contamination of groundwater for given coordinates of pumps and sinks. Graphically, proposed approach is presented in fig. 1.

For comparison, as the benchmark, the NSGA-II [1] algorithm has been used which is one of the most commonly and widely used evolutionary algorithm for multiobjective optimization. It is based on non-dominated sorting procedure ensuring that the better the individual is (it is non-dominated, or dominated only once (by non-dominated individuals) etc.) it is the higher probability that the individual is directly carried over to the next generation. The algorithm is just a state-of-the-art in the multiobjective optimization field and is constantly improved (recently NSGA-III algorithm has been proposed which is non-dominated sorting based algorithm for solving many-objective optimization problems [16]).

4.1 IGA-FEM solver

Recently, the isogeometric finite element method (IGA-FEM) [8] became the state-of-the-art method for performing accurate simulations of difficult time dependent problems. This is be-

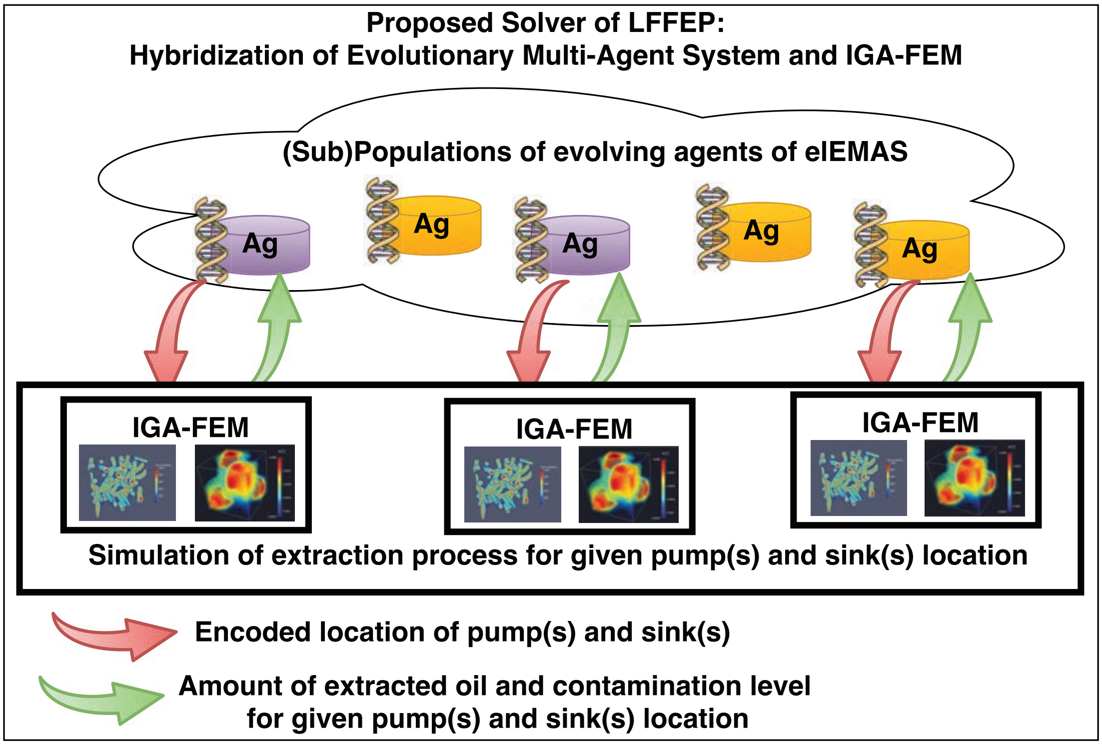


Figure 1: Proposed solver of LFFEP: hybridization of NSGA-II and IGA-FEM

cause the IGA-FEM utilizes B-splines and non-uniform rational B-splines (NURBS) [9] as basis functions and thus provide a global higher continuity of the numerical solution, especially when the simulated physical phenomena requires higher order partial differential equations (PDE). Additionally there are linear computational cost solvers allowing for fast accurate solution of the system of linear equations obtained from IGA-FEM simulations [10, 11]. That is why the IGA-FEM solver has been used as the primal problem solver modeling the process of pumping the water into oil/gas bearing formations as the non-linear flow in heterogeneous media.

In our IGA-FEM simulation each time step involves solving an L2 projection problem, as shown in sec. 2, i.e. solving a system of linear equations where the matrix is the Gram matrix of chosen basis functions. For this purpose we use ADI method [10, 11].

Linear computational cost of Alternating Direction Solver is an efficient alternative to classical multi-frontal solvers, delivering $O(N^2p^3)$ computational cost for 3D problems when running in sequential [26], and this cost can be reduced down to $O(N^(4/3)p^2)$ when using parallel shared or distributed memory implementations [27, 28].

The ADI method has been originally introduced in [4, 5, 6, 7] to solve parabolic, hyperbolic and elliptic partial differential equations. Recently, the method has been extended to isogeometric finite element methods simulations. Its sequential implementation delivers linear computational cost with respect to mesh size [10, 11] and the parallel version scales well up to 1,000 processors [14]. The method has been applied as a fast solver to 2D non-stationary problem [10], as well as the preconditioner for ILUPCG iterative solvers in case of solution of non-stationary PDE over complex geometries [11].

The method exploits a special structure of Gram matrix of our basis functions, which are constructed as a tensor product of one-dimensional basic B-splines, i.e. basis functions are defined as:

$$B_{ijk}(x, y, z) = B_i(x)B_j(y)B_k(z) \tag{16}$$

where B_α are one-dimensional basic B-splines. This is possible due to simplicity of domain geometry and boundary conditions. It can be shown that Gram matrix of such basis can be expressed as a tensor product of Gram matrices of one-dimensional bases:

$$M = M_x \otimes M_y \otimes M_z \tag{17}$$

This property allows us to reduce the problem of solving the system of $Mx = b$ to solving multiple systems with smaller matrices M_x, M_y and M_z, which can be done efficiently (in linear time with respect to the number of unknowns), since they are banded. Detailed exposition and full derivation of the algorithm can be found in [13].

The solver for the primal problem has been developed in the frame of the PRELUDIUM grant DEC-2014/15/N/ST6/04662.

5 Experimental studies

Medium permeability map K_q used in the simulation is presented in Figure 3. Most of the medium has constant low permeability (transparent on the picture), except for some highly permeable „conduits". Initial state $u_0(\mathbf{x})$ based on medium permeability map K_q. Let us denote by $\tilde{K}_q$ a function similar to K_q, but assuming values $[0, 1]$ instead of $[1, 1000]$, i.e.

$$\tilde{K}_q(\mathbf{x}) = \frac{K_q(\mathbf{x}) - 1}{1000 - 1} \tag{18}$$

Then u_0 is given by

$$u_0(\mathbf{x}) = 0.1\tilde{K}_q(\mathbf{x})\,\theta_{0.2,0.3}(\|\mathbf{x} - \mathbf{c}\|) \tag{19}$$

where $\mathbf{c} = (0.5, 0.5, 0.5)$.

For the simulation we use $10 \times 10 \times 10$ mesh on a domain cube. We use the time step of order of magnitude $\Delta t = 10^{-7}$ due to stability constraints arising from Courant-Friedrichs-Lewy (CFL) condition.

Assuming above initial conditions the task has been defined as looking for the positions of three pumps and a one sink to minimize groundwater contamination and maximize drained liquid for an oil concentrated in a ball in the center of a cubic domain.

5.1 Experimental results

	Best final HV	Mean HV	Std. Dev.	Friedman test rank
NSGA-II	0.756	0.751	0.0047	2.0
EMAS	0.745	0.664	0.081	1.0

Table 1: Selected comparative characteristics obtained during the experiments

We have performed several independent executions of IGA-FEM solver hybridized with EMAS adjusted for solving the LFFEP problem. Just for comparisons we have also performed several executions of IGA-FEM solver hybridized with NSGA-II algorithm. We used NSGA-II

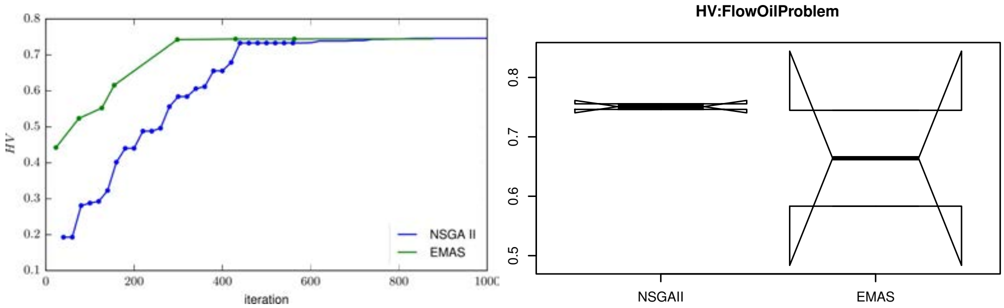
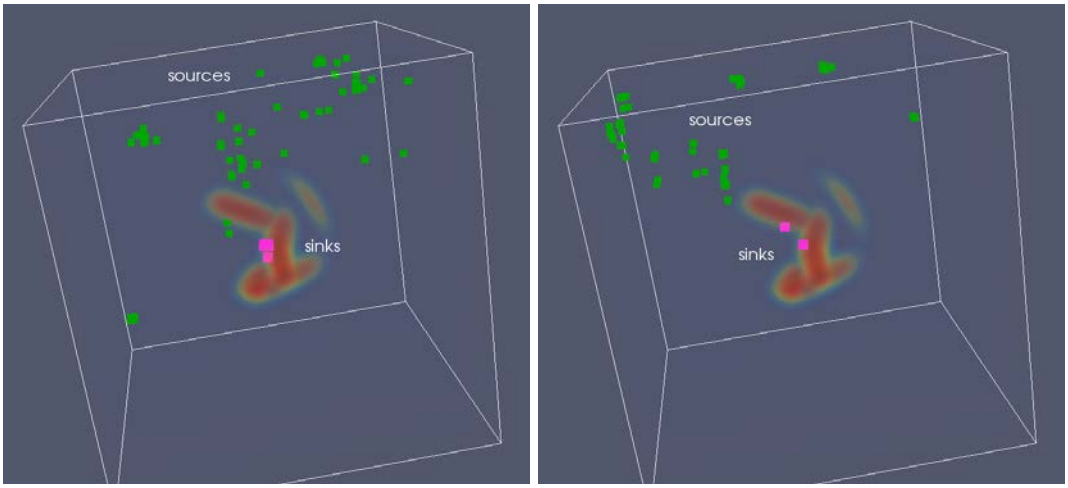


Figure 2: The sample characteristics (a) and the average values (b) of the hypervolume metrics obtained during the experiments

implementation available in jMetal framework [15] and EMAS has been implemented as a new heuristic within jMetal framework. In the consequence both compared metaheuristics used exactly the same computational environment, the same quality indicators, variation operators (for instance crossover, mutation etc.) what makes the comparison reliable and credible. During all experiments the maximum number of fitness function evaluations equals to 1,000 has been defined as a stop condition.



(a) NSGA II (b) EMAS

Figure 3: Location of pumps and sinks in Pareto-optimal solutions found (initial oil pressure distribution displayed in the background)

As a quality measure the hypervolume (HV) [1] metrics has been used (HVR [1] can not be used since the true Pareto frontier is not known).

The sample characteristics and the average values of the hypervolume metrics obtained during the experiments by NSGA-II and EMAS algorithms are presented in figure 2.

As one may see in figure 2a, generally there were such experiments where the characteristic of the HV metrics in the consecutive steps as well as its final value were almost the same (or very similar). In numbers it is confirmed in Table 1. In the first column the best final value of the HV metrics is presented and it is almost the same in both cases. But the general conclusion coming from the preliminary experiments is that NSGA-II algorithm is more stable. It is confirmed both in Table 1 and (graphically) in figure 2b where the mean value of the HV metrics as well as its standard deviation is better in the case of NSGA-II algorithm. It comes from the fact that sometimes EMAS-based solver got stuck what disallowed to obtain a really high-quality solutions since in the NSGA-II based solver the stagnation has never been observed.

From the LFFEP problem point of view it can be interesting where in fact the pumps and sinks should be located according to the obtained non-dominated solutions to maximize the amount of the gas drilled and to minimize the contamination of the groundwater. The sample localization of the pumps and sinks obtained by both i.e. EMAS-based and NSGA-II based solvers are presented in the figure 3. As one may see according to the intuition some optimal sink positions are close to the center of the deposit but there are also some solutions with sinks significantly distant from the center.

One may ask: what in fact is the level of the improvement we can obtain thanks to optimizing the exploitation process according the LFFEP problem defined and using solvers presented in this paper. According to the obtained results the optimization process allows us to decrease the pollution by about 4×10^{-5} (from $\approx 1.9 \times 10^{-5}$ to $\approx 1.5 \times 10^{-5}$). Furthermore, solutions from the final population have quadruple the oil drain in comparison to the best ones from the initial population (increase from $\approx 2 \times 10^{-9}$ to almost 8×10^{-9}).

6 Conclusion

In the paper we have shown that the use of EMAS-based optimization together with IGA-FEM solver for modeling of non-stationary flow of the non-linear fluid in heterogeneous media might be a good solution for solving the real difficult problem of extracting liquid fossil fuels. We have considered one of exploitation methods which consists in pumping chemicals to the geological formation and 'sucking out' the fuel that is pushed out by the solution. A real problem in this case is natural environment contamination caused mainly by chemicals soaking through the geological formations to groundwater. Thus the optimization problem is a multicriteria one with simulation-based objectives evaluation, which makes it difficult to use traditional approaches.

Reported results of computational experiments proved that the proposed models and methods allow for obtaining valuable results and seems to be very promising for further investigation and research. Agent-based reverse problem solver allows to obtain even better results comparing to state-of-the-art NSGA-II algoriithm, but sometimes seems to get stuck in some local optima. The future works will thus include (but won't be reduced to) working on mechanisms preventing premature EMAS stagnation as well as developing three dimensional version of the model of non-linear flow in heterogeneous media, investigating the relation between the IGA-FEM mesh size, and the accuracy of the primal and inverse problem solution and performing extensive experimental studies in supercomputing facilities.

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